# **IT-Security**

# **Chapter 3: Symmetric Integrity Protection**

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### **Overview**



### **Intuition for Data Integrity protection**

#### • Manipulation of messages sent over an insecure network cannot be prevented

- Anyone between the communicating entities can change the message
  - Flip bits, delete bits, replace messages with other ones

#### Encryption schemes do typically NOT enable detection of such manipulations

See the many examples in the exercises

### Data integrity protection mechanisms aim at detecting any message manipulation by unauthorized entities

- Can be realized in form of Modification Detection Codes (MDCs)
- Can be realized in form of Message Authentication Codes (MACs)

### **Idea of Modification Detection Codes**



### Idea of Message Authentication Codes



- Alice and Bob share a secret key
- Alice computes MAC of message using key
- Alice sends message and MAC to Bob
- Attacker may change message and/or MAC
- Bob computes MAC of received message using key
- Compares computed MAC to received MAC
- Decides that message was received as sent if both are the same

### **Hash Function**

#### • A hash function is a function h with the properties

- compression: h maps an input x of arbitrary bitlength to an output h(x) of fixed bit-length n
- ease of computation: given h and x, h(x) is easy to compute
  - there is a polynomial-time algorithm to compute h(x)

#### • A collision of a hash function is a

▶ pair of inputs  $x_1, x_2$ , with  $h(x_1) = h(x_2)$ 



### Minimal Number of Collisions of a hash function

#### • Basic pigeonhole principle

• If *n* pigeonholes are occupied by n + 1 pigeons

then at least one pigeonhole is occupied with more than one pigeons

- Generalization
  - If *n* pigeonholes are occupied by  $k \cdot n + 1$  pigeons

then at lease one pigeonhole is occupied with more than k pigeons



#### Consequence for the minimal number of collisions

• If a hash function maps  $k \cdot n$  messages to n hash values

then there is at least one hash value to which k or more messages hash

• E.g., if n = 16, and  $k \cdot n$  = 64, then there are 4 or more messages that hash to the same value

### **Cryptographic Hash Function**

#### • A hash function is preimage resistant

if given a randomly chosen y = h(x) but not x it is
 computationally infeasible to find any pre-image x' with h(x') = y

#### A hash function is second preimage resistant

- if given x, h(x) it is computationally infeasible to find a second preimage x' ≠ x with h(x') = h(x)
- A hash function is collision resistant
  - ▶ if it is computationally infeasible to find a pair x, x' with  $x' \neq x$  and h(x') = h(x)

Computationally infeasible here means theoretically computable but impractical (except with negligible probability) as it takes too many resources and too much time to compute!

A cryptographic hash function is a preimage resistant and collision resistant hash function

### **Relations between the Properties**

• Collision resistance  $\Rightarrow 2^{nd}$  pre-image resistance

⇒ A cryptographic hash function is always
 2<sup>nd</sup> pre-image resistant as it is collision resistant

- 2<sup>nd</sup> pre-image resistance ⇒ collision resistance
- Collision resistance ⇒ pre-image resistance
- Pre-image resistance ⇒ collision resistance
- 2<sup>nd</sup> pre-image resistance *⇒* pre-image resistance
- Pre-image resistance ⇒ 2<sup>nd</sup> pre-image resistance

Note that some of these implications do hold for a narrower definition of a hash function mapping long fixed lengthmessages to much shorter hashes

### **Example Proof of the Relations**

• Collision resistance  $\Rightarrow 2^{nd}$  pre-image resistance

#### **Proof by contradiction**

- Assume h is collision resistant but not 2<sup>nd</sup> pre-image resistant, then given x, h(x) we can find an x' such that
  - h(x') = h(x).
- Thus, we have found the collision (x, x')
- ▶ This contradicts our assumption which thus cannot hold

### **Example Proof of the Relations**

**Collision resistance** *⇒* **pre-image resistance** 

#### **Constructive proof**

- Assume g is collision resistant n-bit hash function
- Define  $h(x) = \begin{cases} 1 \parallel x & \text{if the bitlength of } x \text{ is } n \\ 0 \parallel g(x) & \text{otherwiese} \end{cases}$
- Then h(x) is a (n + 1)-bit hash function that is collision resistant but not pre-image resistant

Note that  $a \parallel b$  stands for the concatenation of two bit-strings a and b

A similar proof can be used to proof that 2<sup>nd</sup> -pre-image resistance does not imply pre-image resistance

### **Related Terms and Synonyms**

#### • Cryptographic hash function = Secure hash function

- pre-image resistant + collision resistant
- thereby also second-preimage resistant
- One way hash function
  - pre-image resistant
- Second preimage resistant = weak collision resistant
  - as it is implied by collision resistant
- Collision resistant = strong collision resistant
- Output of hash function = hash value = message digest = hash

### Ideal Hash Function through Random Oracle Model

#### • An ideal *n*-bit hash function *h* would operate as follows

- ▶ Upon receipt of a message *m* it has not seen before
  - Pick an *n*-bit value uniformly at random from  $\{0,1\}^n$  and return it as h(m)
- ▶ Upon receipt of a message *m* it has seen before
  - Return the same value h(m), that was picked when the message was new



- We can thus use it to determine an upper bound on
  - how pre-image resistant a real-world hash function can be
  - how collision resistant a real-world hash function can be



### **Complexity of Attacks against Ideal Hash Function**

#### Pre-image attack: Given a hash value y

- Randomly select x and compute h(x)
- Compare h(x) to y
  - Stop if h(x) = y
  - Return to Step 1 otherwise
- Requires  $0.69 \cdot 2^n = O(2^n)$  hash computations to

```
find a pre-image with probability \frac{1}{2}
```

#### **Collision attack:**

- Randomly select x and compute h(x), store result
- Compare each newly computed hash with the values already stored
  - Stop if h(x) = h(x') and output (x, x')
  - Return to Step 1 otherwise
- Requires 1. 18 ·  $2^{n/2} = O(2^{n/2})$  hash computations to find a collision with probability  $\frac{1}{2}$

• Both statements on the complexities can be proven by the solution to flavors of the so-called Birthday Problem

### **Example Proof of Complexity of Pre-image Attack**

#### The 1<sup>st</sup> birthday problem

- Given N different balls in a jar and one fixed ball  $\hat{x}$
- How many times do we need to pull from the jar independently and uniformly at random with put back until with probability P we pulled  $\hat{x}$  at least once?

#### Solution

- ▶ If we chose one ball x, then the probability that  $x \neq \hat{x}$  is  $1 \frac{1}{N}$
- The probability that we are unsuccessful k-times in a row is  $(1 \frac{1}{N})^k$
- The probability P that we picked  $\hat{x}$  at least once if we pick k-times is thus

$$P = 1 - (1 - \frac{1}{N})^k \sim 1 - e^{-\frac{k}{n}}$$
 (using the approximation  $1 - x \sim e^{-x}$  ( $x \ll 1$ ))

• Thus  $k \sim \ln[1/(1-P)] N$  and in particular for  $\mathbf{P} = \frac{1}{2}$  we get  $\mathbf{k} \sim \mathbf{0}.69 \cdot \mathbf{N}$ 



### Similar but Omitted: Proof of Complexity of Collision Attack

#### **Birthday Paradoxon**

- ▶ Given *N* different balls in a jar
- How many times do we need to pull independently and uniformly at random with put back from the jar until with probability P we drew the same ball  $\hat{x}$  twice?

#### Solution

- We need to draw  $k \sim \sqrt{2 \ln[1/(1-P)] N}$  times and in particular for **P** =
  - $\frac{1}{2}$  we get k ~ 1. 18  $\cdot \sqrt{N}$  = 1.18  $\cdot N^{\frac{1}{2}}$





### **Examples for Hash Functions and their Properties**

Algorithm	Maximum Message Size in Bit	Block Size in Bit	Rounds	Size of Hash Value	Year
MD5	2 <sup>64</sup>	512	64	128	1991
SHA-1	2 <sup>64</sup>	512	80	160	1993
SHA-2-224	2 <sup>64</sup>	512	64	224	2002
SHA-2-256	2 <sup>64</sup>	512	64	256	
SHA-2-384	2 <sup>128</sup>	1024	80	384	
SHA-2-512	2 <sup>128</sup>	1024	80	512	
SHA-3-256	unlimited	1088	24	256	2015
SHA-3-512	unlimited	576	24	512	

- MD5 and SHA-1 are not considered collision resistant anymore and should no longer be used
- SHA-2 not broken yet, but break needs to be feared

### **Example Time-Lines of Breaks of MD5 and SHA-1**

#### MD5

- 1993: Collision found by Boer and Bosselaers
- 1996: Attack that found a collision in a modified version of MD5
- 2004: Wang et al. found collisions in MD5 and others
- 2005: Further make collision finding feasible on a laptop (8 hours to find a collision)
- 2006: Black et al. implemented a toolkit for collisions in MD5
- 2007: Stevens et al. find collisions in less than 10 seconds on a on a 2.6Ghz Pentium 4
- 2009: MD5 attacks successfully used to fake certificates
- March 2011 IETF recommendation: MD5 should not be used any more where collision resistance is needed

#### SHA-1

- 2004: 2<sup>nd</sup> preimage attack on SHA-1 in 2<sup>106</sup>
- 2005: Attack found by Wang et al. that finds a collision with 2<sup>69</sup> hash operations
- 2013: Attack by Stevens et al. finds identical prefix collision in 2<sup>61</sup> and chosen prefix collision in 2<sup>77.1</sup>
- 2015: Attack by Stevens et al. that finds a Free-Start Collision on 76-step SHA-1 in 2<sup>50</sup> hash operations
- 2017: Collision on SHA-1 found
- 2016/2017 SHA1 was phased out starting from 2016/17 by all major browsers
- SHA-1 is not used anymore in the context of certificates

### **Overview**



- Intuition
- More formal definition

#### • Message Authentication Codes

- Based on cryptographic hash functions
- Based on symmetric ciphers
- Combining Encryption and Integrity Protection
- Based on cryptographic hash functions
  - Based on symmetric ciphers



How can we get integrity protection?



How can integrity protection be attacked

How can we securely combine encryption and integrity Protection

### **Modification Detection Codes**



### Message Authentication Codes



- MACs require a secret key as additional input
- MAC functions can be constructed from cryptographic hash functions or block ciphers

### **Definition of a Message Authentication Code**

• A Message Authentication Code (MAC) is a family of functions MAC<sub>K</sub> parameterized by a secret key

#### K with the following properties

- **Ease of computation** given K and x, MAC<sub>K</sub>(x) is easy to compute
- Compression MAC<sub>K</sub> maps an input x of arbitrary finite bit-length to an output MAC<sub>K</sub>(x) of fixed bit-length n
- Computation resistance for every *K* and any given number of pairs  $(x_i, MAC_K(x_i))$  it is without knowledge of *K* computationally infeasible to compute any pair  $(x, MAC_K(x))$  with *x* different from all  $x_i$ 
  - Note that such pairs $(x_i, MAC_K(x_i))$  can typically be obtained by an attacker by eavesdropping
- MACs can be constructed from cryptographic hash functions or block ciphers

### HMAC: Bellare, Canetti, and Krawczyk 1996

• Let h be a cryptographic hash function, then for a message M and key K

 $\operatorname{HMAC}_{K}(M) = h(K \oplus \operatorname{opad} \parallel h(K \oplus \operatorname{ipad} \parallel M))$ 

#### where opad and ipad are constant values.

- ipad = 0x36....0x36
- opad = 0x5C...0x5C

• HMAC is computation resistant if h is cryptographic hash function

- HMAC construction does not introduce any new risk
- ▶ ipad and opad guarantee that different keys are used in the inner and outer hash computation
  - The two keys will differ in half of the bits because of the choice of ipad and opad

### Can't we just use $h(K \parallel M)$ as MAC?

- Unfortunately, no! Simple constructions like that are typically insecure
- Many hash functions (e.g., MD2, SHA-1, SHA-2) operate on blocks of M
  - $\blacktriangleright M = M_0 \parallel M_1 \parallel \dots \parallel M_n$
  - h operates on the first block  $M_0$  which is then used as first state to operate on  $M_1$ ,...
  - Thus, h(M) is the initial state of  $h(M \parallel X)$ 
    - I.e., from known hashes of shorter messages, we can construct hashes of longer messages
  - ▶ I.e., knowing  $h(K \parallel M)$  we can compute  $h(K \parallel M \parallel X)$  without knowing the key

### CMAC: Constructing a MAC from a Block Cipher

- CMAC uses a block cipher  $E_K$  of block length b = 64 or b = 128
- A message M is split into n blocks of length b:

 $M = M_1 \parallel M_2 \parallel \ldots \parallel M_n$ 

- If the last block  $M_n$  is not of length b it is padded with  $10 \dots 0$  until it is b bit long
- CMAC computation is equivalent to
  - ► Applying CBC Mode of encryption to the message with an IV of all zeros
  - Except that the last block is additionally masked with
    - A sub-key K<sub>1</sub> if M<sub>n</sub> is of bit length b and with
    - A sub-key K<sub>2</sub> if M<sub>n</sub> was padded to be of full bit length b
  - ▶ The resulting last ciphertext block is the CMAC of the message

### Illustration of the CMAC Computation

#### If M<sub>n</sub> has block length b



If M<sub>n</sub> is padded to b bits





M<sub>n</sub>

Eκ

 $K_1$ 

CMAC(M)

#### K1 and K2 are derived from K

- L= E<sub>K</sub>(0<sup>b</sup>), where 0<sup>b</sup> is the bitstrings of b zeros
- $\blacktriangleright R_{128} = 0^{120} 10000111$
- ▶  $R_{64} = 0^{59} 11011$
- Then K<sub>1</sub> is computed by
  - ▶ If MSB1(L) = 0, K<sub>1</sub> = L<<1</p>
  - Else  $K_1 = L \bigoplus R_b$
- K<sub>2</sub> is computed by
  - If MSB1(K<sub>1</sub>) = 0, K<sub>2</sub> = K<sub>1</sub><<1</p>
  - ► Else  $K_2 = (K_1 << 1) \oplus R_b$

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 $M_{n-1}$ 

Eκ

### **Rational for the Two Different Keys**

- Let's assume we have a one block message M = 011
  - ▶ then  $CMAC_K(M) = E_K(01110 \dots 0 \oplus K_2)$
- The one block message  $M' = 01110 \dots 0$  has  $CMAC_K(M') = E_K(01110 \dots 0 \oplus K_1)$
- So, if  $K_1$  and  $K_2$  were the same,
  - then  $CMAC_K(M)$  would be the same as  $CMAC_K(M')$
  - ▶ Thus, an attacker could replace *M* with *M*' without the receiver noticing it

### Why Do we need the Masking with $K_1$ and $K_2$

- Using a "pure" CBC-MAC is insecure!
  - ▶ I.e., without the masking by  $K_1$  or  $K_2$  in the last step
- A CBC-MAC allows for forgery in some specific settings
  - ▶ For example, let *M* and *P* be two one-block messages and *MAC<sub>K</sub>* be a CBC-MAC
    - $MAC_K(M) = E_K(M)$
    - $MAC_K(P) = E_K(P)$
  - If an attacker observes  $M, MAC_K(M)$  and  $P, MAC_K(P)$ 
    - he can forge a valid CBC-MAC on  $M \parallel (P \oplus MAC_K(M))$  without knowing K because:
    - $MAC_{K}(M \parallel (P \oplus MAC_{K}(M))) = E_{K}(E_{K}(M) \oplus P \oplus MAC_{K}(M)) = E_{K}(P \oplus MAC_{K}(M) \oplus MAC_{K}(M)) = E_{K}(P) = MAC_{K}(P)$
- The masking with  $K_1$  and  $K_2$  solves this problem

### **Replay Protection**

#### • A MAC computed over a message alone

does not protect against replay of the protected message

#### Replay protection requires additional input

- Make a message sent twice distinguishable from a replayed message
- Additional input
  - Counters
    - Time stamps
    - Sequence numbers (SQN)
  - ▶ Random numbers as challenges (*RAND*)



# **Replay Protection**

	Advantage	Disadvantage	Main Use
Timestamps	No explicit initial value needs to be known by sender and receiver	Require time synchronization between sender and receiver	Whenever sender and receiver are time- synchronized anyway
SQNs	Simple, no time- synchronization required	Requires (re-)synchronization of SQN, Agreement on initial value, Window of acceptable SQNs if in-order delivery of messages cannot be guaranteed	Protect all traffic between two entities once keys are established
RAND	Does not need synchronization, requires random number generator	Requires receiver to challenge the sender and thus adds communication overhead	Mainly used as part of authentication and key agreement protocols, where single messages need to be protected against replay

### **Overview**



### **Combining Integrity Protection and Encryption**

Encrypt, then MAC:  $E_{K_1}(M) \parallel MAC_{K_2}(E_{K_1}(M))$ 

- Encrypt plaintext with K<sub>1</sub>
- Compute MAC on encrypted plaintext with *K*<sub>2</sub>

#### **MAC, then Encrypt:** $E_{K_1}(M \parallel MAC_{K_2}(M))$

- Encrypt plaintext with K<sub>1</sub>
- Compute MAC on encrypted plaintext with *K*<sub>2</sub>
- MAC can only be checked AFTER decryption

**Encrypt and MAC:**  $E_{K_1}(M) \parallel MAC_{K_2}(M)$ 

- Encrypt plaintext with *K*<sub>1</sub>
- Compute MAC on plaintext with K<sub>2</sub>
- MAC may reveal information on M
- MAC can only be checked AFTER decryption

#### Special authenticated modes of encryption

- E.g., Galois Counter Mode (GCM)
- E.g., Counter mode with CBC MAC (CCM)
- Typically take an **encrypt then MAC** approach

### **Example: Galois Counter Mode of Encryption (GCM)**

#### • Mode of encryption that also provides integrity protection

- Authenticated Encryption with Associated Data (AEAD) Mode
  - Allows for additional data to be integrity protected but not encrypted
- Based on a block cipher with 128-bit blocklength
- GCM can be used as MAC alone
  - called GMAC then
- Properties
  - ► Can use IVs of arbitrary length
  - Easy to implement very efficiently in hardware
  - Very good software performance

Data blocks to protect

 $A_1 \parallel \dots \parallel A_m \parallel P_1 \parallel \dots \parallel P_n$   $A_i \ (i = 1, \dots, m) \text{ are to be}$ integrity protected only  $P_i \ (i = 1, \dots, n) \text{ are to be integrity}$ protected and encrypted

### Illustration of GCM Encryption and Integrity Protection Operation



# Data blocks to protect $A_1 \parallel P_1 \parallel P_2 \parallel P_3$ $A_1$ integrity protected $P_i (i = 1, ..., 3)$ integrity protected and encrypted $Y_0$ Initial counter value $Y_i = Y_{i-1} + 1$ $H = E_K(0^{128})$

• = Multiplication in GF(2<sup>128</sup>)

## Illustration of GCM Decryption and Integrity Verification Operation



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### **GCM in Formulars**

Data to be protected

$$M = A_1 \parallel \ldots \parallel A_m \parallel P_1 \parallel \ldots \parallel P_n$$

#### Initialization:

 $Y_0$  Initial counter value

$$Y_i = Y_{i-1} + 1$$
  
 $H = E_K(0^{128})$  wheer  $0^{128} = \underbrace{0 \cdots 0}^{128}$ 

• = Multiplication in GF(2<sup>128</sup>)

Encryption:  $C_i = E_K(Y_i) \oplus P_i$  for (i = 1, ..., n)Integrity Protection:  $T_0 = 0$   $T_i = (T_{i-1} \oplus A_i) \bullet H$  for i = 1, ..., m  $T_{m+i} = (T_{m+i-1} \oplus C_i) \bullet H$  for i = 1, ..., n  $T_{m+n+1} = (T_{m+n} \oplus (len(A) \parallel len(C))) \bullet H$  $GMAC_K(M) = T_{m+n+1} \oplus E_K(Y_0)$ 



Note: if  $P_n$  is not of full block length, then  $C_n$  is not of full block length If  $A_m$  or  $C_n$  are not of full block length, they are padded with zeros in the *GMAC* computation

### **Reminder: Multiplication in GF(2128)**

- $GF(2^{128})$  is the finite field with  $2^{128}$  elements
  - It is unique up to isomorphism
- GCM uses the irreducible polynomial  $f(x) = 1 + x + x^2 + x^7 + x^{128}$
- Identify each 128-bit string  $a = a_0 \dots a_{127}$  with the polynomial  $a(x) = \sum_{i=0}^{127} a_i x^i$
- Multiplication of a and b in GF(2<sup>128</sup>) is then defined as
  - bit string representation of  $a(x) \cdot b(x) \mod f$ :

 $(\sum_{i=0}^{127} a_i x^i) \cdot (\sum_{i=0}^{127} b_i x^i) \mod f$ 

### **Summary**

#### Message Authentication Codes provide integrity protection

- MACs can be constructed from cryptographic hash functions: HMAC
- MACs can be constructed from block ciphers: CMAC
- Simple constructions like  $h(M \parallel K)$  or CBC-MAC are insecure

#### Cryptographic hash functions

- Are pre-image resistant and collision resistant
- Finding a pre-image with probability  $\frac{1}{2}$  requires at most  $O(2^n)$  hash computations for an ideal hash function
- Finding a second pre-image with prob.  $\frac{1}{2}$  requires at most  $O(2^n)$  hash computations
- Finding a collision with prob.  $\frac{1}{2}$  requires at most at most  $O(2^{n/2})$  hash computations

#### • Replay protection requires additional input to an integrity protection mechanism

E.g., a counter, a time stamp, or a random number selected by the receiver



### **Summary**

#### • Securely combining encryption and integrity protection

- Requires an encrypt-then-MAC type of an approach
  - Special modes of encryption which also provide integrity protection use this as well
- Other approaches are insecure or unnecessarily expensive

#### • The GCM Mode of encryption is an example for an AEAD cipher

- Provides encryption and integrity protection
- Makes use of CTR mode for encryption
- ► Can additionally protect the integrity of data which is not encrypted



### References

#### • Johannes Buchmann, Einführung in die Kryptographie, Springer Verlag 2016

- Chapter 11 on Hash Functions and Message Authentication Codes
- W. Stallings, Cryptography and Network Security: Principles and Practice, 8<sup>th</sup> edition, Pearson 2022
  - Chapters 12: Message Authentication Codes

#### Specifications

- ▶ HMAC: NIST Specification FIPS 198-1
  - https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.198-1.pdf
- ► CMAC:
- GCM and GMAC NIST Special Publication 800-38D
  - https://nvlpubs.nist.gov/nistpubs/Legacy/SP/nistspecialpublication800-38d.pdf